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Group 1

I affirm that I will not give or receive any unauthorised help on this exam, and that all work will be my own.

Problem 1

(a) $G = (V_N, V_T, S, P)$

$V_N = \{S, A, B, C, D, E\}$

$V_T = \{a\}$

$S = S$

$P = \{S \to ACaB,\ Ca \to aaC,\ CB \to DB \mid E,\ aD \to Da,$
$AD \to AC,\ aE \to Ea,\ AE \to \lambda\}$

S
$A\underline{Ca}B$
$Aa a\,\underline{CB}$
$Aaa\,DB \mid E$

$Aa\underline{aD}B$
$A\underline{aD}aB$
$\underline{AD}aaB$
$A\underline{Ca}aB$
$Aaa\underline{Ca}B$
$AaaaaCB$

$Aa\underline{aE}$
$A\underline{aE}a$
$\underline{AE}aa$
$\boxed{aa}$

Cojocaru Dan
Group 1

$A\,aaaa\,CB$
$A\,aaaa\,DB$ $\rightarrow$ $A\,aaaa\,E$
$A\,aaaEa$
$A\,aaEaa$
$A\,aEaaa$
$A\,Eaaaa$
$\boxed{aaaa}$

$\Longrightarrow L(G)=\{(aa)^n \mid n \geq 1\}$

or

$L(G)=\{a^{2n} \mid n \geq 1\}$

(b) The production rule for S is $ACaB$ $\Rightarrow$ neither left-linear or right-linear $\Rightarrow$ NOT type 3.

There exist a production rule for aE $\Rightarrow$ not just one non-terminal $\Rightarrow$
$\Rightarrow$ NOT type 2.

Therefore, it is type 1.

Chomsky hierarchy on page 10.

Cojocaru
Dan
Group 1

Problem 2.

$G_2 = (V_N, V_T, S, P)$

$V_N = \{X_0, X_1, X_2\}$

$V_T = \{A, B, \dots, Z\}$

$S = X_0$

$P = \{X_0 \to E X_1, X_1 \to N X_2, X_2 \to D\}$

$L_2(G_2) = \{END\}$

$G_3 = (V_{N_3}, V_{T_3}, S_3, P_3)$

$V_{N_3} = \{S_3, S, A, B, C, D, E, X_0, X_1, X_2\}$

$V_{T_3} = \{a, A, B, \dots, Z\}$

$S = S_3$

$P = \{S_3 \to S / X_0,$
$S \to ACaB,\ Ca \to aaC,\ CB \to DB/E,\ aD \to Da,\ AD \to AC,$
$aE \to Ea,\ AE \to \lambda,\ X_0 \to EX_1,\ X_1 \to NX_2,\ X_2 \to D\}$

$L_1 \cup L_2 = \{END$ or $a^{2n} \mid n \geq 1\}$

Since L_1 is type 1 and L_3 is type 3,
$L1 \cup L_2$ is type 1 (the lowest of the two).

Cojocaru Dan
Group 1

Problem 3. $L = \emptyset$

Grammar:

$G = (V_N, V_T, S, P)$

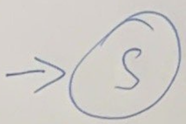
$V_N = \{S\}$

$V_T = \emptyset$

$S = S$

$P = \{S \to 3\}$ (G accepts a symbol not part of the grammar)

FA:



(finite automaton with no accepting states)

Problem 4:

(a) $DFA = \{S, \Sigma, s_0, F, \delta\}$

S - set of states

Σ - input symbols / alphabet

s_0 - starting state

F - set of accepting states

δ - transition function

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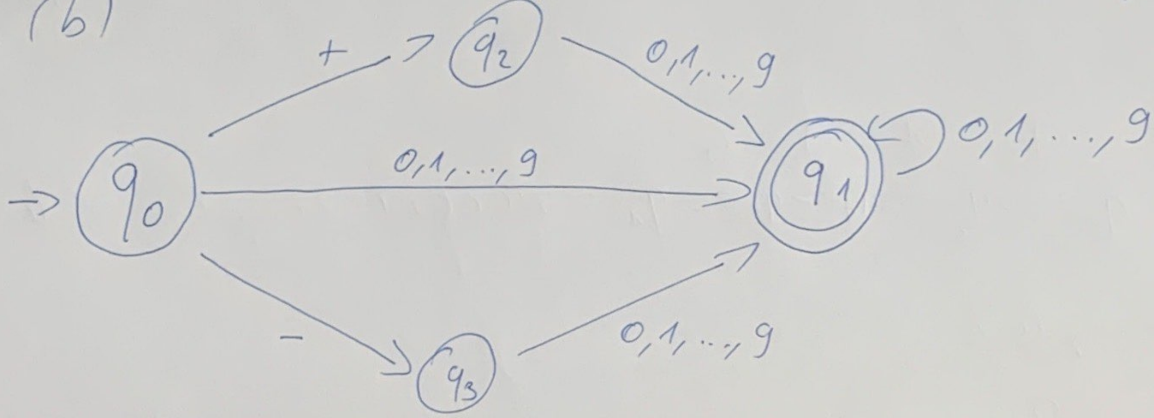
Group 1

Problem 5:

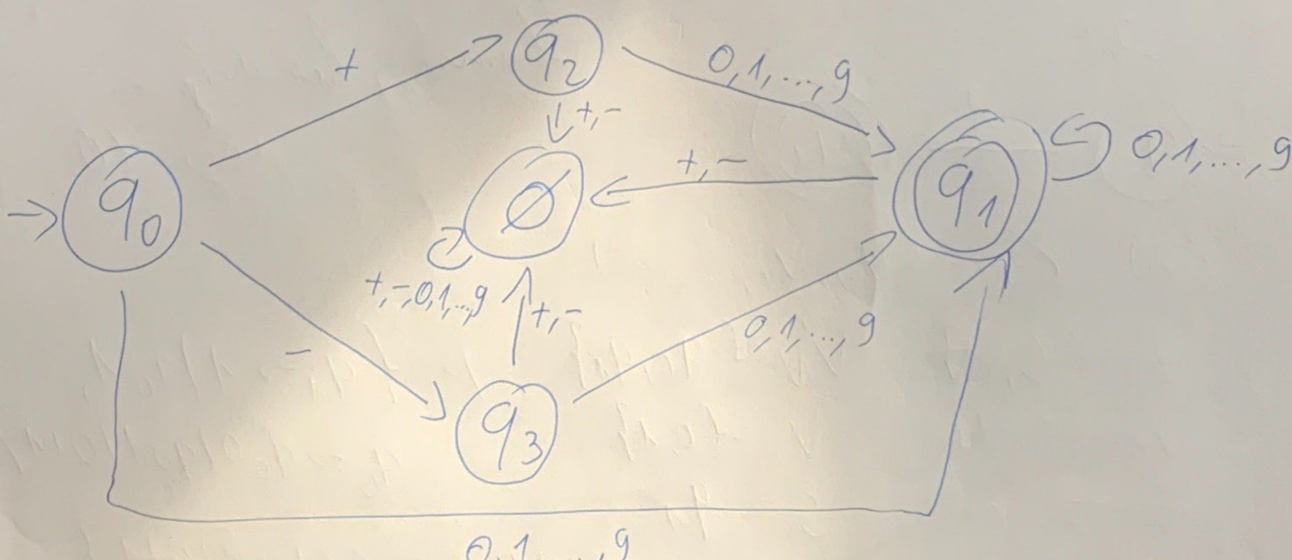
(b)



(c)

	+	−	0,1,...,9
→q_0	q_2	q_3	q_1
q_1			q_1
q_2			q_1
q_3			q_1

	+	−	0,1,...,9
→q_0	q_2	q_3	q_1
q_1	$\emptyset$	$\emptyset$	q_1
q_2	$\emptyset$	$\emptyset$	q_1
q_3	$\emptyset$	$\emptyset$	q_1
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$



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Problem 4: (d)

Example 1:

$q_0 \xrightarrow{+} q_2 \xrightarrow{1} q_1 \xrightarrow{9} q_1 \xrightarrow{6} q_1$ ACCEPT, accepting state

Example 2:

$q_0 \xrightarrow{2} q_1$ ACCEPT, q_1 accepting state

Counterexample:

$q_0 \xrightarrow{+} q_2 \xrightarrow{3} q_1 \xrightarrow{5} q_1 \xrightarrow{-} \emptyset \xrightarrow{2} \emptyset \xrightarrow{4} \emptyset$ NOT ACCEPT

Problem 5: (a)

$L = \{w \mid w$ is a binary string ending in $1\}$

Chomsky hierarchy on page 10.

(b) $G = (V_N, V_T, S, P)$

$\cancel{V_N = \{S, A\}}$ $\quad V_N = \{q_0, q_1\}$ $\quad P = \{q_1 \to 1 \mid q_0 1,$

$\cancel{V_T = \{0, 1\}}$ $\quad V_T = \{0, 1\}$ $\quad q_0 \to q_0 0 \mid q_0 1 \mid 0 \mid 1\}$

$\cancel{S = S}$ $\quad S = q_1$

$\cancel{P = \{S \to 0 \quad S \to A1 \mid 1, A \to A0 \mid A1 \mid 0 \mid 1\}}$

Problem 6:

Cojocaru Dan

Group 1

$G_F = (V_{N_F}, V_{T_F}, S_F, P_F)$

$V_{N_F} = \{q_{0F}, q_{1F}, q_{2F}\}$

$V_{TF} = \{D, a, n\}$

$S_F = q_{0F}$

$P_F = \{ q_{0F} \to D q_{1F},$
$q_{1F} \to a q_{2F},$
$q_{2F} \to n \}$

$G_L = (V_{N_L}, V_{T_L}, S_L, P_L)$

$V_{N_L} = \{q_{0L}, q_{1L}, \dots, q_{7L}\}$

$V_{TL} = \{c, o, j, e, a\ r u\}$

$S_L = q_{0F}$

$P_L = \{ q_{0L} \to c q_{1L},$
$q_{1L} \to o q_{2L},$
$q_{2L} \to j q_{3L},$
$q_{3L} \to o q_{4L},$
$q_{4L} \to c q_{5L},$
$q_{5L} \to a q_{6L},$
$q_{6L} \to r q_{7L},$
$q_{7L} \to u \}$

~~$q_{8L} \to$~~

$G_E = (V_{NE}, V_{TE}, S_E, P_E)$

$V_{NE} = \{q_{0E}, q_{1E}, \dots, q_{22E}\}$

$V_{TE} = \{a, b, \dots, z, @, -, ., 0\}$

$S_E = q_{0E}$

$P_E = \{ q_{0E} \to d q_{1E}, q_{1E} \to a q_{2E}, q_{2E} \to n q_{3E}, q_{3E} \to . q_{4E},$
$q_{4E} \to c q_{5E}, q_{5E} \to o q_{6E}, q_{6E} \to j q_{7E}, q_{7E} \to o q_{8E},$
$q_{8E} \to c q_{9E}, q_{9E} \to a q_{10E}, q_{10E} \to r q_{11E}, q_{11E} \to u q_{12E},$
$q_{12E} \to 0 q_{13E}, q_{13E} \to 0 q_{14E}, q_{14E} \to @ q_{15E}, q_{15E} \to e q_{16E},$
$q_{16E} \to - q_{17E}, q_{17E} \to u q_{18E}, q_{18E} \to v q_{19E}, q_{19E} \to t q_{20E},$
$q_{20E} \to . q_{21E}, q_{21E} \to r q_{22E}, q_{22E} \to o \}$

Problem 6

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$G_C = (V_{NC}, V_{TC}, S_C, P_C)$

$V_{NC} = \{S, q_{0F}, q_{1F}, q_{2F}, q_{0L}, \dots, q_{7L}, q_{0E}, \dots, q_{22E}\}$

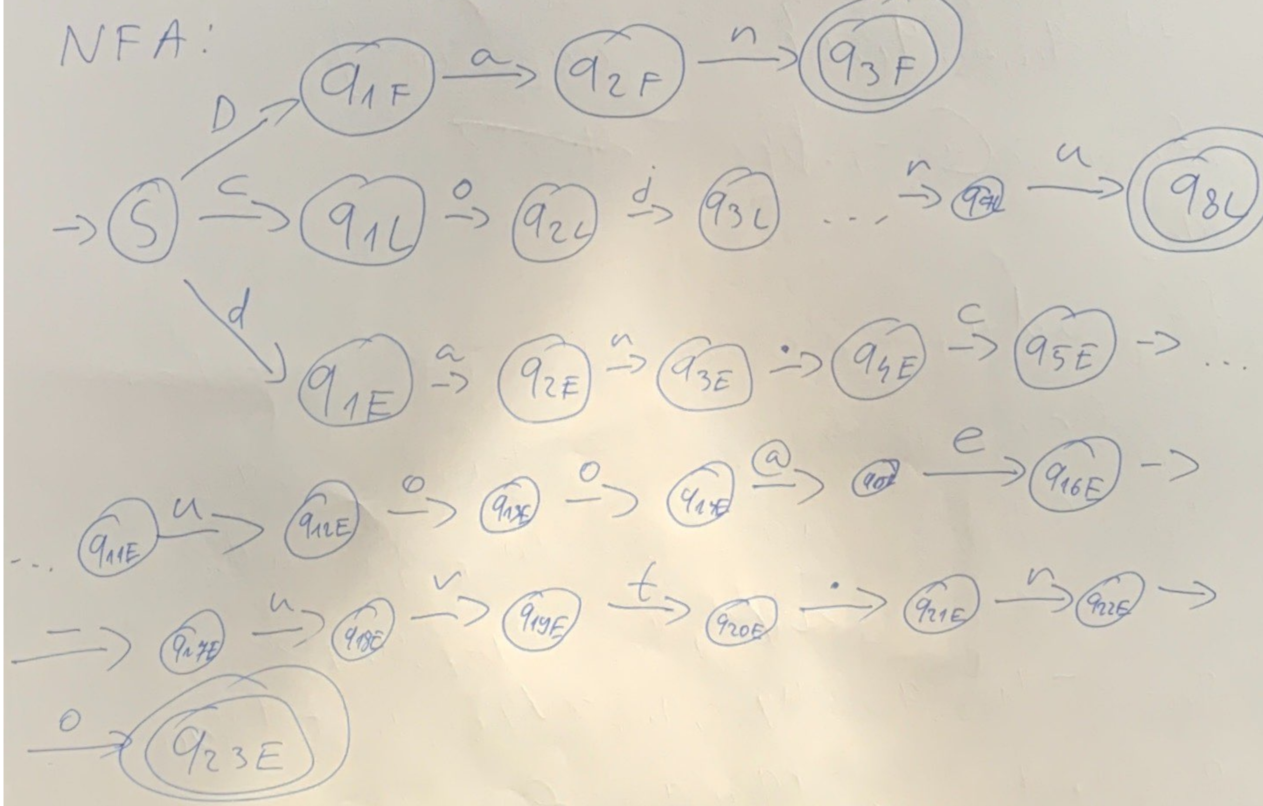
$V_{TC} = \{D, C, a, b, \dots, z, @, -, ., 0\}$

$S_C = S$

$P_C = \{S \to q_{0F} \mid q_{0L} \mid q_{0E}$, all production rules from $P_F, P_L, P_E\}$

The grammar and languages are of type 3 (right-linear). Chomsky hierarchy on page 10.

NFA:



Problem 6

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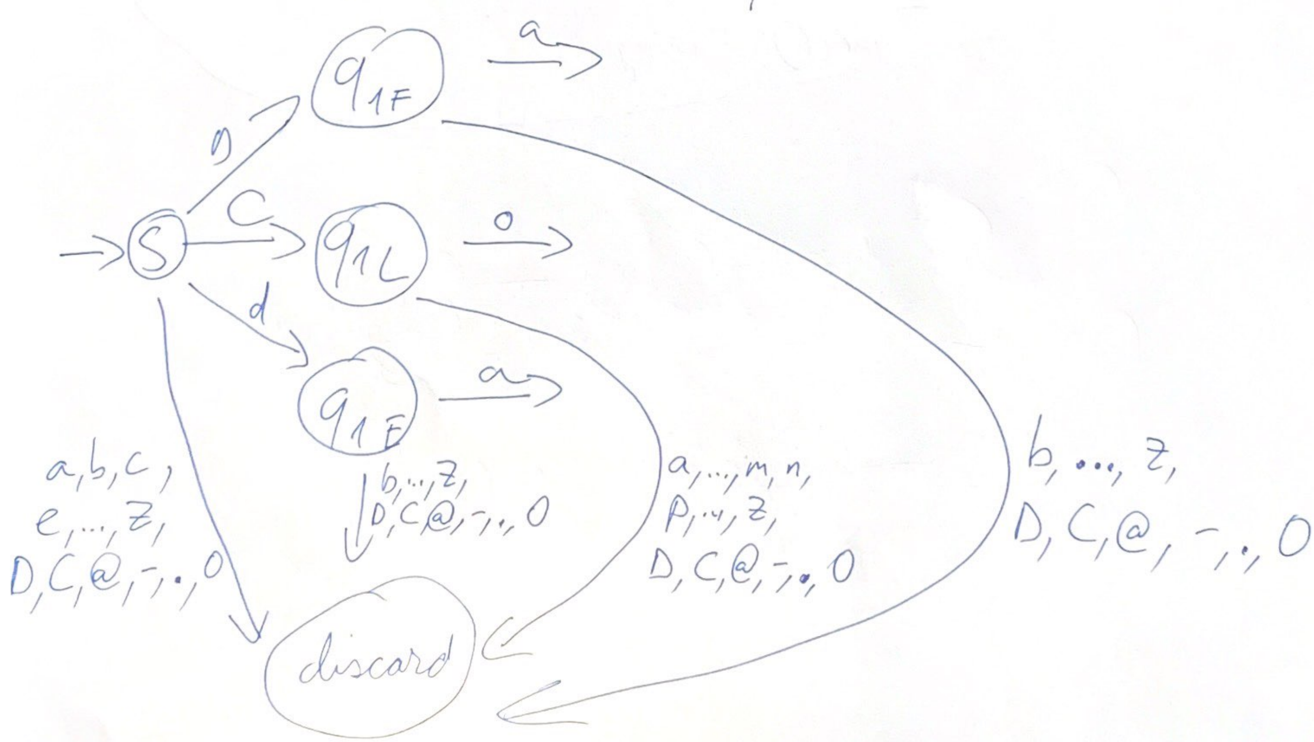
Since the NFA is directly generated from a right-linear grammar, the only thing left to do is to add a discard state which is not final, and to add transitions from each state of the NFA to the discard state, the transitions having the symbols of all unused symbols.

For example, for the first 4 states:



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